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Technical Comments

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Aircraft *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Reply to Comment on "Analytical Model for the Impulse of Single-Cycle Pulse Detonation Tube" by W. H. Heiser and D. T. Pratt

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WE acknowledge the importance of developing bounding estimates based on fundamental thermodynamic concepts. Further, we agree that an approach like that adopted by Heiser and Pratt[1] (referred to as H&P in what follows), which focused on state transformations, is a useful idea that is complementary to the usual flow path analysis. However, we feel that there are aspects of this that should be further examined by the propulsion community. Our remarks and response to the comments of H&P are intended to promote critical thinking about cycle analysis for unsteady propulsion systems.

1) A key issue that must be dealt with in any pulse detonation engine (PDE) analysis is the role of unsteady flow in converting thermal energy into impulse. It is in the treatment of this issue, not the relative merits of state transformation vs flow path analysis, that we differ from H&P's approach. In the case of steady flow, it is sufficient to consider the total enthalpy $h + u^2/2$ (where h is the enthalpy and u the flow velocity) and the usual idealized isentropic compressions and expansions to accomplish the conversion between thermal and kinetic energy. This is the standard approach used in flow path analysis of airbreathing systems such as turbines, ramjets, and scramjets. For these conventional steady flow propulsion systems, energy conservation and known entropy changes uniquely determine the exit velocity, and it is possible to focus on an interpretation based solely on thermodynamic variables.

However, in the case of unsteady flow, the conversion of thermal energy into impulse is not uniquely determined by the thermodynamic state changes. This means that energy balance statements for the total energy $e + u^2/2$ (where e is the internal energy) must be considered and that the unsteady conversion of thermal energy into impulse, including wave propagation processes, must be computed on a case-by-case basis. A thermodynamic cycle for the unsteady detonation process is analyzed on this basis by Fickett and Davis.[2] Applying this cycle to propulsion requires further analysis[3] to treat the fluid mechanics of the combustion product expansion process and the transfer of impulse to the engine. An example of explicitly including the wave processes in a propulsion system model is Foa's[4] method of characteristics analysis of the valved pulsed jet. In the case of internal combustion engines, the Otto and diesel cycles are indeed analyzed with a state approach using only thermodynamic variables, most commonly pressure-specific volume (P, V) or temperature entropy (T, S). In those examples, the energy balance statement is usually simplified by neglecting the kinetic energy of the gas, and the efficiency of the engine can be bounded without any consideration of the fluid mechanics. However, for an arbitrary unsteady flow process, we do not know of any method to uniquely and rigorously use a sequence of thermodynamic states alone to define an upper bound for the conversion of thermal energy into gas momentum and net force on the engine.

2) We recognize that H&P's calculation of cycle efficiency based on thermodynamic cycle analysis is formally correct. However, we disagree with the use of this analysis to compute the performance of unsteady propulsion systems because their model appears to be equivalent to a steady flow analysis. This conclusion is based on two simple observations drawn from studying their presentation: a) there are no time averages or derivatives, unsteady control volume balances, or any energy equations involving unsteady flow work terms in the analysis, and b) the key link between thermal efficiency and propulsive performance [Eq. (8) in H&P] is the standard propulsion textbook result obtained from the steady flow conservation equations.

A key assumption in H&P's analysis is that the thermal efficiency obtained from the thermodynamic state diagram of Fig. 2 in H&P can be equated to the thermal efficiency of a steady flow process between states 0 and 10, and, further, that the thrust is equal to the thrust of an engine with a steady outflow at state 10 and steady inflow at state 0. This approach calculates performance from the entropy increments for each process based on the thermodynamic cycle alone and is described by Foa[4] as the entropy method. The method is based on the fact that, for steady flow, the difference in kinetic energy between the freestream and the exit plane is equal to the work done by the fluid. As Foa[4] points out, this method is applicable only to the analysis of steady propulsive flows or the special case of "square-wave" time dependence. In addition, the mass and momentum contributions of the fuel input must be negligible compared with the corresponding contributions of the airflow (e.g., the fuel-air mass ratio $f \ll 1$), and the average exit flow must be pressure matched. The first condition is usually satisfied for hydrogen- and hydrocarbon-air combustion, but the second condition is not universally satisfied for PDEs.

Applying the entropy method to an arbitrary unsteady flow requires reconsidering the basis of this technique. To do this, we analyze an unsteady, but periodic, propulsion system during steady flight with a steady inflow and an unsteady outflow. The performance is calculated using the control volume of Fig. 1 and the unsteady equations for mass, momentum, and energy conservation.[5] Assuming that the engine operates in a cyclic mode (no storage terms during steady flight), we average those equations over a cycle

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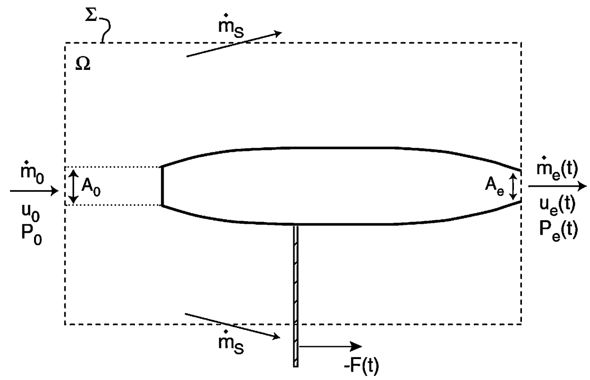

Fig. 1 General unsteady propulsion system with control volume used for performance calculation.

to obtain an expression for the average thrust

$$\bar{F} = \frac{1}{\tau}\int_0^{\tau} F(t)\,\mathrm{d}t = \overline{\dot{m}_e(t)u_e(t)} - \dot{m}_0 u_0 + A_e[\overline{P_e(t)} - P_0] \quad (1)$$

and the average energy balance

$$\overline{\dot{m}_e(t)h_{te}(t)} = \dot{m}_0 h_{t0} + \overline{\dot{m}_f(t)} \cdot Q \quad (2)$$

where t is the time, τ is the cycle time, $\dot{m}$ represents the mass flow rate, u the flow velocity, P the pressure, h_t the total enthalpy, A_e the exit plane area, $\dot{m}_f$ the fuel mass flow rate, and Q the heat release per unit mass of fuel. Subscripts 0 and e denote quantities in the freestream and at the exit plane, respectively, and $\overline{()}$ denotes temporal average over a cycle. For arbitrarily unsteady outflow these equations do not reduce to the standard[5] steady flow results. The cycle averages of the terms $\dot{m}_e(t)u_e(t)$, $\dot{m}_e(t)h_{te}(t)$ and P_e will depend on the details of each parameters's time dependence at the engine exit. Even if the exit is pressure matched on average ($\overline{P_e} = P_0$), this does not mean that the thrust is optimized or uniquely determined by the time-averaged exit plane properties.

To circumvent this difficulty, Foa[4] extends the entropy method to unsteady flows by redefining the average of an exit plane property X as

$$\langle X_e \rangle = \frac{1}{\tau \overline{\dot{m}_e}}\int_0^{\tau} \dot{m}_e(t) X_e(t)\,\mathrm{d}t \quad (3)$$

Using this averaging method and assuming the conditions for use of the entropy method are satisfied (i.e., $f \ll 1$ and $\overline{P_e} = P_0$), we rewrite Eqs. (1) and (2) as

$$\bar{F} = \dot{m}_0(\langle u_e \rangle - u_0) \quad (4)$$

and

$$\langle h_{te} \rangle = h_{t0} + \bar{f} Q \quad (5)$$

defining $\bar{f}$ as $\overline{\dot{m}_f}/\dot{m}_0$. These equations are now analogous in form to the equations used in the steady flow entropy method. Foa then defines the thermal efficiency for unsteady flows as

$$\eta_{\mathrm{th}} = \frac{\langle u_e^2 \rangle - u_o^2}{2\bar{f}Q} \quad (6)$$

and, using the energy conservation equation (5), this is equivalent to

$$\eta_{\mathrm{th}} = 1 - \frac{\langle h_e \rangle - h_0}{\bar{f}Q} \quad (7)$$

This is the desired extension to the steady flow result of the entropy method. To calculate performance from Eqs. (4–7), the averages $\langle u_e \rangle$, $\langle u_e^2 \rangle$, and $\langle h_e \rangle$ have to be calculated. One obvious difference from the conventional steady flow analysis is that generally $\langle u_e \rangle^2 \neq \langle u_e^2 \rangle$. Foa suggests that these differences can be taken into account by defining an efficiency of nonuniformity

$$\eta_v = \langle u_e \rangle^2 \big/ \langle u_e^2 \rangle \quad (8)$$

By definition, η_v is less than one and equals one only when the exhaust flow is steady or a square-wave function of time. The cycle-averaged thrust can be written as

$$\bar{F} = \dot{m}_0\left[\sqrt{\eta_v\left(2\eta_{\mathrm{th}}\bar{f}Q + u_o^2\right)} - u_0\right] \quad (9)$$

A more subtle but equally important issue is that η_{th} will not generally have the same value as the corresponding steady flow quantity and there is no obvious relationship between the two. The values of η_v and η_{th} must be computed from detailed experimental measurements, unsteady analytical models, or numerical simulations. In conclusion, the steady flow entropy method as used by H&P is rigorously applicable only to propulsion systems for which the exhaust flow is either steady or a square-wave function of time. This treatment of the exhaust flow is not applicable to the situation of a detonation tube that we have considered[6] and, in general, will not apply to most PDE concepts.

Even as a purely steady flow analysis, there are unresolved issues with the H&P approach. This can be shown by comparing their study with a flow path analysis for a steady detonation ramjet engine.[7–9] The flow path studies show that state 3 is uniquely related to the Chapman–Jouguet condition and cannot be selected arbitrarily as is done in Fig. 2 of H&P. There are strict constraints[7–9] on the possible choices for state 3 because of the requirements for detonation wave stabilization. If the value of the temperature ratio Ψ is too low, then solutions to the Chapman–Jouguet conditions do not exist. These flow path-related constraints are not captured by the H&P analysis and explain the substantial difference observed between the performance predictions for a steady flow detonation ramjet[7–9] and those based solely on thermodynamic cycle analysis.

The values of 3000–5000 s given in our paper[6] for the fuel specific impulse refer to two estimates: the value of about 3000 s given in Fig. 7 of H&P, and higher values given in Ref. 3 in our paper (not 2, as incorrectly referenced) of up to 4000 s for hydrocarbons[10] and 8000 s for hydrogen.[10] H&P are correct that a value of 2800–3600 s is appropriate for their analysis.

The issue of nozzle effectiveness that H&P raise provides an opportunity to highlight the importance of unsteady flow. Substantial specific impulse increases can be observed with nozzles but for a different reason than might be anticipated from steady flow notions. In particular, adding a simple straight extension filled with air can also increase the specific impulse.[3] This has been observed both in single-cycle[3,11] and multicycle[12] static experiments. This effect has been analyzed[13,14] and shown to be a purely unsteady gasdynamic effect. The experimental data,[3,11,12] the model of Cooper and Shepherd,[3] and the numerical simulations of Li et al.[13] and Li and Kailasanath[14] for partial filling show that the fuel-based specific impulse can increase by more than three times from the value for a fully filled detonation tube when the tube fraction filled with combustible gas is decreased sufficiently. This means that, in the static case, a specific impulse of up to 4750 s can be reached for stoichiometric JP10-air based on experimental data for a fully filled tube.[15] This value is at least 30% higher than the predicted values of 2800–3600 s from the thermodynamic cycle analysis of H&P. Clearly, this effect is completely outside the scope of any steady analysis and points out the difficulty of creating bounding estimates for PDE performance based solely on a thermodynamic cycle.

Just as in steady flow nozzles, the ratio of effective chamber pressure (Chapman–Jouguet pressure or state 3 pressure) to environment pressure plays the key role in determining the effectiveness of nozzles in improving performance. Numerical simulations for a pulse detonation rocket engine[16] have shown that an optimized constrained converging-diverging nozzle can provide only marginal benefit in specific impulse (about 3%) for low pressure ratios, whereas a significant performance gain is obtained at high pressure ratios (up to 53% increase in specific impulse). In these simulations, the angles of the converging and diverging parts of the nozzle were fixed at 14 deg. Hence, increasing the length of the nozzle increases the nozzle expansion ratio, and the benefits of the tamping effect at low pressure ratios are mitigated by flow overexpansion. We expect the optimization results will be a strong function of the nozzle angle, unlike the case of steady flows. These results and the unsteady flow effects mentioned earlier (the case of the partially filled detonation tube) highlight the complexity of predicting the effects of nozzles on performance.

3) We did not intend for the use of the word "fictitious" to be derogatory—only to indicate that the sequence of states is assumed rather than derived from any experimental observation, detailed analysis, or simulation. It might turn out to be a very valuable fiction if the crucial issue of unsteady energy conversion can be resolved in a simple manner. We agree that many of the observations made in paragraph two of our conclusions are generic to a variety of propulsion systems on the basis of dimensional analysis alone.

References

[1]Heiser, W. H., and Pratt, D. T., "Thermodynamic Cycle Analysis of Pulse Detonation Engines," *Journal of Propulsion and Power*, Vol. 18, No. 1, 2002, pp. 68–76.

[2]Fickett, W., and Davis, W. C., *Detonation Theory and Experiment*, Dover, New York, 2001, Chap. 2, pp. 35–39.

[3]Cooper, M., and Shepherd, J. E., "The Effect of Nozzles and Extensions on Detonation Tube Performance," AIAA Paper 2002-3628, July 2002.

[4]Foa, J. V., *Elements of Flight Propulsion*, Wiley, New York, 1960, Chaps. 13–15.

[5]Hill, P. G., and Peterson, C. R., *Mechanics and Thermodynamics of Propulsion*, 2nd ed., Addison Wesley, Longman, Reading, MA, 1992, Chap. 5, pp. 146–149.

[6]Wintenberger, E., Austin, J. M., Cooper, M., Jackson, S. I., and Shepherd, J. E., "An Analytical Model for the Impulse of a Single-cycle Pulse Detonation Tube," *Journal of Propulsion and Power*, Vol. 19, No. 1, 2003, pp. 22–38.

[7]Wintenberger, E., and Shepherd, J. E., "The Performance of Steady Detonation Engines," AIAA Paper 2003-0714, 2003; also *Journal of Propulsion and Power* (submitted for publication).

[8]Dunlap, R., Brehm, R. L., and Nicholls, J. A., "A Preliminary Study of the Application of Steady-State Detonative Combustion to a Reaction Engine," *Jet Propulsion*, Vol. 28, No. 7, 1958, pp. 451–456.

[9]Sargent, W. H., and Gross, R. A., "Detonation Wave Hypersonic Ramjet," *ARS Journal*, Vol. 30, No. 6, 1960, pp. 543–549.

[10]Bussing, T. R. A., Bratkovich, T. E., and Hinkey, J. B., "Practical Implementation of Pulse Detonation Engines," AIAA Paper 97-2748, July 1997.

[11]Falempin, F., Bouchaud, D., Forrat, B., Desbordes, D., and Daniau, E., "Pulsed Detonation Engine Possible Application to Low Cost Tactical Missile and to Space Launcher," AIAA Paper 2001-3815, July 2001.

[12]Schauer, F., Stutrud, J., and Bradley, R., "Detonation Initiation Studies and Performance Results for Pulsed Detonation Engine Applications," AIAA Paper 2001-1129, July 2001.

[13]Li, C., Kailasanath, K., and Patnaik, G., "A Numerical Study of Flow Field Evolution in a Pulsed Detonation Engine," AIAA Paper 2000-0314, Jan. 2000.

[14]Li, C., and Kailasanath, K., "A Numerical Study of Reactive Flows in Pulse Detonation Engines," AIAA Paper 2001-3933, July 2001.

[15]Wintenberger, E., Austin, J. M., Cooper, M., Jackson, S., and Shepherd, J. E., "Impulse of a Single-Pulse Detonation Tube," Graduate Aeronautical Lab., California Inst. of Technology, Report FM00-8, Pasadena, Aug. 2002.

[16]Morris, C., "Quasi-One-Dimensional Modeling of Pulse Detonation Rocket Engines," AIAA Paper 2003-5204, July 2003.